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# Manifestations and meanings of cognitive conflict among mathematics students in Embu, Kenya

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**Establishing how cognitive conflict is manifested by students is an important first step in understanding how teachers can utilize cognitive conflict to improve students' learning experiences. This paper presents findings from the analysis of qualitative data drawn from a larger study that explored the role of cognitive conflict in promoting students' conceptual development in mathematics. The study participants were secondary school mathematics students and their teachers drawn from twenty-five public secondary schools in Embu West Sub-County in Kenya. Data were gathered through surveys and semi-structured interviews. The interviews were transcribed and coded, followed by organization of the codes into categories that were used to develop themes. The findings indicate that students experienced cognitive conflict in three significant ways: a moment to (co) construct one's mathematical meaning, confusion as a result of teacher's behaviorist stance, and a fleeting moment of magic. The paper recommends that teachers should take advantage of cognitive conflict as a strategy for scaffolding mathematics learning by giving students tasks that provoke critical thinking so that as students work on the tasks, their naïve understandings of the concepts are challenged.**

**Key words:** Cognitive conflict, manifestations, mathematics, meanings, students learning.

## INTRODUCTION

Cognitive conflict is a widely recognized important factor in the process of conceptual change and can be effectively utilized as a teaching-learning strategy to promote the conceptual development of students (Mufit et al., 2018). The notion of cognitive conflict has recently been receiving much attention in teaching and learning, particularly in the area of mathematics education. There is evidence in the mathematics education literature that significant numbers of students are often confronted with contradictions between their way of describing and explaining concepts and the way such concepts are explained by their peers, teachers, or textbooks. As a result, there is currently great interest among mathematics educators in the issue of conceptual change through the use of cognitive conflict, as it has been found to be instrumental in promoting deeper learning and conceptual understanding in mathematics (Adnyani, 2020; Watson, 2002, 2007).

Cognitive conflict refers to a situation where a student is confronted with a discrepancy between their existing cognitive elements (such as attitudes, perceptions, knowledge, and behaviors) and new information or idea

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(Waxer and Morton, 2012; Zazkis and Chernoff, 2006). During mathematics learning, cognitive conflict occurs when students have a preconceived idea about how a mathematical problem should be solved which differs from the way it is being solved (Maume and Mathews, 2000). Cognitive conflict can be viewed as a learning strategy that is interactive, inspiring, fun, and challenging to students (Lee and Kwon, 2001). In mathematics education, cognitive conflict has been observed by several researchers as a situation that can play an important role in students' acquisition of mathematical concepts that can also act as evidence of mathematics learning (Baddock and Bucat, 2008; Fraser, 2007; Lee et al., 2003; Maharani and Subanji, 2018; Sayce, 2010; Susilawati et al., 2017; Tall, 1977; Zazkis and Chernoff, 2006).

Utilizing cognitive conflict during mathematics lessons can help promote the idea of Cognitively Guided Instruction (CGI) which underscores the need for teachers to pay more attention to students during mathematics learning to improve students' thinking as well as teacher skills in explaining concepts (Jacobs et al., 2007). Also, teachers are encouraged to use Cognitive Acceleration through Science Education (CASE) and Cognitive Acceleration through Mathematics Education (CAME) as programs that help to scaffold mathematics learning based on cognitive conflict (Swan et al., 2005). Furthermore, teachers need to appreciate and understand students' existing ideas and understandings, and to present students with situations that provoke cognitive conflict to reveal the inadequacy of the students' ideas and to encourage the formation of new knowledge.

Mathematics teaching based on cognitive conflict can as well improve students' ability to solve mathematical problems, develop critical thinking, and improve their communication skills (Gal, 2019; Hermkes et al., 2018; Putra et al., 2019; Webb et al., 2019; Widada et al., 2018). However, the use of cognitive conflict as a teaching-learning strategy has not been sufficiently studied in the area of mathematics education. The few studies in this line have emphasized the need for mathematics teachers to possess knowledge about cognitive conflict and its role in mathematics teaching and learning. With this knowledge, the teachers will be better placed to scaffold students learning and to improve students' critical reasoning skills (Li, 2019; Makonye and Khanyile, 2015).

As noted by some researchers, there is a need to extend and deepen teachers' professional understanding of teaching and learning practices and strategies that support students' conceptual understanding (O'Brien and Iannone, 2018). Furthermore, there is a need for research on teaching-learning strategies that improve students' achievement in mathematics (Cho et al., 2015; Tinto, 2012). Indeed, researchers have also underscored the need for an investigation into how teachers modify their teaching methods towards responding to the needs of their students, as well as on how teachers can incorporate a variety of teaching methods to ensure students' active engagement during the learning process (Murphy et al., 2019; Sutopo, 2014; Webb et al., 2019). This paper seeks to provide valuable insights into how students experience cognitive conflict during mathematics learning to enrich the teachers' strategies for scaffolding student learning of mathematical concepts using cognitive conflict. In particular, the paper reports on the findings from the analysis of qualitative data drawn from a larger study that explored the role of cognitive conflict in promoting students' conceptual development in mathematics. In the following section, a review of relevant literature is presented, followed by a description of the methodology adopted in the study.

## LITERATURE REVIEW

In their review of the literature on cognitive conflict, Lee and Kwon (2001) synthesized the signs that students are likely to exhibit when confronted by cognitive conflict. The signs outlined include anxiety, hesitancy, uneasiness, tension, vacillation, doubt, perplexity, frustration, confusion, and reappraisal of the situation to try and resolve the conflict. Therefore, the teacher needs to check for these signs in students during mathematics teaching and learning with a view to assist them to overcome the cognitive conflict and thereby gain a deeper understanding of the concepts involved (Piaget, 1985; Vygotsky, 1978; Wyrasti et al., 2016). In the absence of such support, cognitive conflict can lead to students experiencing mathematics anxiety, which in turn can lead to low self-esteem and, ultimately, poor performance in the subject (Devine et al., 2018). In addition, mathematics teachers need to acquire competencies on how to utilize cognitive conflict moments to support students learning to guarantee conceptual understanding by students (Ashman and Conway, 2017; Mulungye et al., 2016).

Cognitive conflict occur when students experience a contradiction between their existing knowledge and new experiences during their interaction with peers and teachers (Piaget, 1932). The utility of cognitive conflict in promoting reasoning skills can only be realized if students are supported to reconcile the conflicting ideas by examining, explaining, justifying, and questioning their preconceived ideas and beliefs (Lee and Kwon, 2001; Lee et al., 2003; Peled and Shahbari, 2015; Putra et al., 2019). In the teaching-learning situation, cognitive conflict moments should be viewed as motivational moments to ensure knowledge transfer and skill development in problem-solving (Akpinar et al., 2009; Chow and Treagust, 2013).

The role of cognitive conflict in mathematics teaching and learning has been explored from different perspectives by different researchers. Some researchers have studied the phenomenon based on resolving it in a realistic situation with modeling characteristics as well as its role in developing students' understanding of specific mathematical concepts (Peled and Shahbari, 2015; Watson, 2007). Some other researchers have examined the role of cognitive conflict in improving spatial visualization based on student ability, sources of students' errors, and misconception in algebra (Mulungye et al., 2016; Susilawati et al., 2017). Others have analyzed the characteristics of students' cognitive conflict when solving problems based on information processing theory (Pratiwi et al., 2019). In addition, others have examined the effects of a mathematics cognitive acceleration program on student achievement and motivation (Finau et al, 2018). This paper adds into this repertoire of literature on the role of cognitive conflict in mathematics teaching and learning by considering how cognitive conflict is manifested by students and what meanings are embedded in these manifestations, with a view to inform how teachers can utilize cognitive conflict to improve students' conceptual understanding in mathematics.

## METHODOLOGY

### Research design

This study adopted a mixed methods research design, as it helped in organizing the research data as well as in viewing the phenomenon under study as a whole. This design was appropriate since it allowed the researchers to gather and analyze detailed information from the research participants (Creswell, 2014), and thereby answer the research questions exhaustively. This paper reports the findings from the analysis of qualitative data, as well as the implications of these findings for teachers, educators, and policy makers.

### Research context

The study was carried out in Embu West Sub-County in Embu County, Kenya. In Kenya, there are forty-seven counties, and in each county, there are two categories of secondary schools: public and private. The public secondary schools are further grouped into four categories: National, Extra County, County, and Sub-County schools, depending on students' performance and teaching-learning resources available. The secondary education system in Kenya consists of four grades, called Forms (Form 1, 2, 3, and 4). There are 25 public secondary schools in Embu West Sub-County, 6 of which are single-sex boarding schools and 19 are co-education secondary schools.

### Sampling procedures and sample size

The study used purposive sampling (Creswell, 2013) to select the sub-county and the participants for the study. Form Two students aged between 15 and 16 years were purposively selected because it is a critical stage in secondary school mathematics in Kenya, where students need to be supported as they develop critical thinking and problem-solving skills. It is also the stage at which new mathematical concepts are introduced following the transition from primary school mathematics. The students were purposively selected based on their performance in recent mathematics examination, whereby the top performers were selected since they were found to possess an interest in mathematics and thus their participation was considered crucial in answering the research questions for the study. This is because students who enjoy mathematics always make use of deeper learning strategies, which results in better grades, and vice versa (Pekrun, 2006). The sample size was determined using the Yamane model to arrive at a sample of 350 students from 2800 targeted form two students (Yamane, 1967). To get the exact number of Form Two students who participated in the study per school, the researcher calculated this value depending on the number of Forms Two students in the school relative to the total. For instance, if the total number of Form Two students in a given school was 232 then only 29 students were selected. All teachers of mathematics for form two class or classes in the school participated in the study by giving their valuable information in questionnaires.

### Research methods and instruments

The study used surveys and one-on-one semi-structured interviews to collect research data. The two methods were used as they complemented each other: the surveys allowed gathering of a huge amount of data within a short time, while the interviews allowed in-depth investigation of the phenomenon of interest (Creswell, 2014). An interview guide was used to obtain students' manifestations and meanings of cognitive conflict in mathematics teaching and learning. In addition, questionnaires were used to gather information on participants' demographics as well as their perceptions of the research topic. The questionnaires had both open- and close-ended questions. Based on the methods and instruments discussed, this paper has reported on qualitative information obtained from participants interviewed.

### Pretesting of the research instruments

A pilot study was done to test the validity and reliability of research instruments for the study. The validity of the instruments was prepared by information obtained from different articles online by the researcher in consultation with supervisors. The instruments were piloted in one randomly selected secondary school outside the study area and the school was not included in the actual study sample. The result from the piloting study gave a Cronbach's Alpha of 0.74, which indicated that the instruments were valid to obtain valuable information for the study. The data collected at this stage was useful in the modification of the instruments so that they could yield valuable data needed to answer the research questions.

### Data collection procedures

Approvals to conduct the study were obtained from the National Commission of Science, Technology, and Innovation (NACOSTI) in Kenya. Access to the target schools was approved by the Sub-County Director of Education in Embu County, while permission to contact the teachers and students was granted by the school principals. Initial interactions with the research participants were aimed at creating rapport, explaining to the research participants the purpose of the study, and familiarizing with the research contexts. The research participants were assured of confidentiality and anonymity. At least four students were engaged in a one-on-one semi-structured interview from each school until the point where data saturation was attained; that is, until that point when the participants' responses were not adding any new information to the previous data collected (Bernard, 2012). In total, 68 students (32 females and 36 males) drawn from 17 secondary schools (6 single-sex and 11 mixed-sex) participated in the interviews. The interviews were conducted in a quiet place within the school compound outside the participants' class time. The 20 to 30 min long interviews were audio-recorded and later transcribed. The students' interviews focused on their self-concepts as well as their experiences and meanings of cognitive conflict in mathematics. The teachers' interviews explored the strategies employed by teachers to support students' learning of mathematics as well as the teacher's level of awareness of the strategy of cognitive conflict in teaching mathematics.

#### Data analysis and coding

The interview transcriptions were read and re-read before they were coded following thematic analysis of Terry et al. (2017). The analysis entailed reading and re-reading the whole text and developing codes. The codes were then organized into larger categories and finally, the categories were organized into four overarching themes that are now presented in the subsequent sections. An illustration of how one of the themes was arrived at is shown in Appendix Table 1.

## RESULTS AND DISCUSSION

Through the process of thematic analysis, three themes were identified from the data. These are: a moment to (co) construct one's mathematical meaning, confusion as a result of a teacher's behaviorist stance, and a fleeting moment of magic. These themes are described in the next subsections.

### A moment to (Co) construct one's mathematical meaning

Students' experienced cognitive conflict as opportunities to (co) construct their mathematical knowledge through group discussions so that they can get a better conceptual understanding of contradicting concepts. Besides, some students viewed the cognitive conflict as an opportunity to individually make sense of mathematical concepts by consulting their teachers and peers as well as reference materials. Therefore, this indicates that cognitive conflict strategy made students develop three important values in their character while learning for conceptual development; namely, hardworking, curiosity, and responsibility. During the interviews, some students explained that as they consult their teachers they always found different solutions to ideas that contradicted their long-lived skills during mathematics learning. For instance, a male student argued that they got introduced to some topics in Form Two that he was unable to understand in Form One. Consequently, the student stated that he sought assistance from his peers to understand the concepts. The student said:

I can say that in Form One there were some topics we *were having challenges. For example, Simultaneous Equations, I came to understand this topic just another day in Form Two when I consulted a friend* (Student. 33, 2020).

The student argued that the reason to seek help from the peers as opposed to his mathematics teacher was due to difficulty in understanding his teacher's concept explanation. As observed by Sayce (2010), some of the manifestations of cognitive conflict reported by the students include low self-esteem and stuck moments where students were unable to understand and follow what the teacher was explaining during teaching. During the study, a student argued as follows:

*I just feel low because I may see I am in class, the teacher is teaching and I feel just low because I am unable to understand due to confusion, but at last, I must find help from the students who got the concept from the teacher and finally solved the problem* (Student. 3, 2020).

Students can also react to cognitive conflict by using revision materials available. The students claimed that as they read ahead of the teacher, they experienced cognitive conflict as a result of connecting the method learned from the books and the one which the teacher presented during teaching. During the interview, one of the participants argued that when he encountered cognitive conflict he would consult his peers and as a group, they would consider a simpler method to solve the problem. The student stated that:

*Some of us read ahead of the teacher and you find that if you read ahead of the teacher, the teacher might come with a different formula but the answer is the same, but under that condition, the teacher may understand and leave you with your formula but later the student may be confused on which method to use, but we make use of our group and try to adopt a shorter method so that we can all understand it and use* (Student. 67, 2020).

Cognitive conflict provides opportunities for students to (co)construct their understanding of mathematical concepts. As such, teachers should encourage cooperative learning approaches by providing opportunities for group discussions, and by encouraging students to consult from their peers and the teacher whenever they encounter challenges with problems in mathematics (Makonye and Khanyile, 2015; Sayce, 2010; Webb et al., 2019). Also, teacher educators should equip student-teachers with skills on how to set mathematical tasks that require higher-order reasoning skills so that students can have deeper conceptual understanding and thereby achieve the intended learning outcomes (Bloom and Krathwohl, 1956). Finally, teacher educators should enhance student-teachers with skills in group work so that they can group students based on their vabilities that promote productive work among the

**Step 1:** Write the numbers at the top of the Common Factors Grid, leaving some space to the left of the numbers as well as below the numbers.

**Step 2:** Find the least prime number which is a factor of at least one of the given numbers and write it in the space to the left.

**Step 3:** If the prime number in Step #2 is a factor of the number on the right, then divide the number by the prime and write the quotient below each number.

**Step 4:** Repeat Steps #2 and #3 until the only common factor for the numbers is 1.

**Box 1. Division method of determining LCM.**

groups.

**Confusion as a result of the teacher's behaviorist stance**

Behaviorists view learners as passive individuals and, hence, teachers who subscribe to this approach tend to make the lesson teacher-centered (Pange et al, 2010). This study found that the teachers embraced behaviorism in teaching mathematics by not considering students' methods. This is because the students argued that teachers failed to articulate their ways of solving mathematical problems during teaching. As a result, the students found it difficult to follow the teacher's methods and felt confused when instructed to use such methods in problem-solving. Some students also reported that they at times experienced difficulties when they attempted solving mathematical problems using the teacher's prescribed methods. For example, in one instance, a student explained that in applying the teacher's method, they were required to look for a smaller fraction, which was confusing as compared to their method. During the interview, the student explained that the teacher's method of solving problems in *Ratio and Proportions* made him get more confused because he would get stuck when trying to apply the teacher's method. The student argued as follows:

*The teacher forced me to stop using my primary method and use the one he has taught that needs one to hide and search for a smaller value and multiply that with the other. In the process of using the teacher's method, I get confused and get stuck on what I should do next* (Student. 63, 2020).

**The students also described the discrepancy they noticed between the procedure for determining the Least Common Multiple (LCM) in their primary school (Box 1) and the one used in secondary school (Box 2) as described below.**

During the interviews, a female student claimed that her secondary school teacher regarded the method in Box 1 as a time-consuming method and advised her to better use the one in Box 2. The student stated as follows:

*The teacher told me that this method* (pointing at Box 1) *although it is correct, it is time-consuming and he stressed to me that it is better to use the one he had taught about power forms* (the one in Box 2) (Student. 21, 2020).

The student argued that in applying the method in Box 2 she always experienced cognitive conflict in cases where the exponents are greater than one to identify the least value of the numbers given. This indicates that the teacher's behaviorist stance can discourage students from exploring diverse ideas while solving problems in mathematics, thereby hindering their mathematics learning. It is therefore important for the teacher to appreciate students' ideas and opinions in the learning process by adopting a constructivist approach (Graffam, 2003; Vosniadou and Verschaffel, 2004). This will help transform cognitive conflict moments into learning moments since students will develop more interest in the concepts for which they experience cognitive conflict. Furthermore, teachers should employ cognitive conflict to facilitate students' mastery of content and guide the students to develop skills for better conceptual understanding (Chambers and Timlin, 2019; Adnyani,

**Step 1:** Find the prime factorization of each number then write it in exponential form.

**Step 2:** For the numbers with a common prime factor base, select the prime number that has the highest power.

**Step 3:** If a distinct prime factor has **NO** matching prime factor base in the list, immediately include this factor with its exponent in the collection of numbers that you will multiply later.

**Step 4:** To determine the Least Common Multiple (LCM), multiply all the numbers that you have collected or gathered from steps #2 and #3.

**Box 2.** Prime factorization method for determining LCM.

2020; Hermkes et al, 2018; Kang et al., 2010; Rahim et al., 2015). There is thus a need for teachers to adopt appropriate pedagogical strategies for supporting students to overcome cognitive conflict in mathematics. Moreover, this study found that students have experience with much of the content covered at lower levels in secondary school from their primary school knowledge but in secondary school, the concepts are the same but only with some variations. Therefore, this paper has emphasized that teacher educators should prepare teachers based on the challenges they are likely to face when they meet high school students, especially at the lower levels.

### A fleeting moment of magic

Some students experienced cognitive conflict as a fleeting moment of magic based on how new strategies of problem-solving were introduced to them by the teacher. The students argued that the teacher made mathematics concepts appear abstract based on how the teacher explained them. Although the teachers viewed fast concept explanation as a strategy for developing problem-solving skills in students, the students indicated that they were uncomfortable with this approach, arguing that the situation did not help them accommodate and use the gained skills. In addition, the students indicated that their teacher needed to link the concepts with the students' experiences for better conceptual understanding. One of the participants said:

*The teacher has guided me in the method there but If I come to calculate the area by trying to separate the figures using the separation method taught, I find it difficult because without capturing the method of the teacher which is different from mine, and it was explained fast, I find it stressful* (Student. 3, 2020).

This student attributed his experiencing of cognitive conflict to the teacher's hasty explanation of the concept. The majority of the students expressed the need for a step-by-step concept explanation by the teacher to facilitate their understanding of the concepts and thereby aid in resolving cognitive conflict. The students further advocated for contextualization of concepts by the teacher to ensure that the students develop conceptual understanding. One student said:

*Yes, the teacher should explain concepts step-by-step so that I can understand the ideas well without getting stuck on the way* (Student. 30, 2020).

Teachers who move fast while explaining mathematical concepts without considering students' experiences sometimes omit important steps that students could utilize in problem-solving. At times, this omission failed to recognize that students had their way of solving the problem. For example, one of the students experienced cognitive conflict simply because the student had his way of solving the problem, which was different from the one the teacher used. During the interview, the student shared a mathematical problem in *Commercial Arithmetic* whose solution during her primary school days differed from the solution in secondary school. The question was framed as follows:

Calculate the profit received by a farmer if he sold his bags for about KShs 1,800 at a 20% profit. The student explained that her primary school mathematics teacher told them that in solving such a problem, they should start by saying that KShs 1,800 = 100%, and then asking themselves the question: what about 120%? She reported that using her primary school knowledge the answer to the question can be calculated as follows:

$$profit = (120 - 100)\%,$$

$$KShs\ 1{,}800 = 100\%, 120\% = ?$$

$$\frac{1{,}800 \times 120}{100},$$

$$= KShs\ (2{,}160 - 1{,}800) = 360.$$

From the above student calculations, it is clear that the student can apply her primary school knowledge in determining profit for the question given. As she proceeded to secondary school, the student reported that her secondary school mathematics teacher solved and instructed them to always solve such mathematical problems as follows:

$$\frac{20}{100} \times 1800 = 360$$

The student revealed that as a result of the teacher's abstract explanation shown above, she experienced cognitive conflict in understanding the new method and failed to know what was happening during teaching.

The sentiments from the students call upon teachers of mathematics to ensure that students are systematically taken through problem-solving to develop better skills in solving mathematical problems. The explanations provided during problem-solving should resonate with students' experiences to ensure that the students do not view the concepts as abstract. However, the findings also indicated that teachers of mathematics should ensure that they do not leave some of the students as they move along, otherwise they will be doing an injustice to the struggling students. Finally, the findings from this study emphasized that teachers should make use of contextualization in mathematics teaching so that students can identify with the school mathematics and thus develop better problem-solving skills.

## SUMMARY OF THE STUDY

This paper has examined students' manifestations and meanings of cognitive conflict in mathematics from the perspective of the research participants. Based on the research findings, three themes were identified to characterize students' experiences of cognitive conflict in mathematics. Several implications for teachers and teacher educators have been identified. In particular, the paper encourages teachers to check for signs of cognitive conflict during the teaching-learning process and to seize such moments to scaffold students' learning. The paper also emphasizes that teachers should provide tasks that promote cooperative learning so that students can learn from each other. The paper has also emphasized the need for the teacher to encourage students to seek help from reference materials, peers, and the teacher to help them build their repertoire of mathematical tools and techniques. As noted by other scholars, cognitive conflict has great potential in promoting conceptual change in mathematics and, as such, teachers and teacher educators should endeavor to orchestrate opportunities of cognitive conflict in mathematics to encourage critical thinking among students.

## RECOMMENDATIONS

This paper recommends further studies to assess the role of gender and cognitive conflict in mathematics. Also, this study supports the argument by Maker (2020) that studies should be done on how education programs can be enriched with strategies that can improve students' performance in Science, Technology, Engineering, and Mathematics (STEM) careers.

## CONFLICT OF INTERESTS

The authors have not declared any conflict of interests.

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**Appendix Table 1.** Thematic analysis process showing how one of the themes was arrived at.

| Codes | Examples of participants quotes | Categories | Theme |
|---|---|---|---|
| Seeking teacher(s) help | I consult the teacher to maybe explain the method to me or use another simpler method | Conceptual development through teacher's guidance | A moment to (co)construct one's mathematical meaning |
| | I do consult the teacher many times and practice more so that I can understand | | |
| | I do consult teachers of mathematics in the school in areas of confusion | | |
| | The ones which are hard and I am unable to solve I go and consult my teacher | | |
| | …when I go to consult I find that the teacher uses a different method from one I know in finding the solution of the sum | | |
| Seeking peer(s) help | …was consulting my friends to solve it | Acquisition of knowledge from peers | |
| | I consult from peers on difficult areas | | |
| | I go and consult my fellow students who know the concepts | | |
| | I consulted my desk mates and my other classmates | | |
| | Many times I consult my fellow students | | |
| Employ reference materials | I use different books and I check on how the question is solved | Use of reference materials to learn | |
| | I got the method in a revision book | | |
| | Mostly I borrow books from the library | | |
| Learn individually | I train on my method | Individual commitment and responsibility | |
| | I practice more and more on it | | |
| | I do keep doing practices continuously until I understand the method | | |
| | I just practice it often by doing some questions | | |
| Group discussion | I do discussions with my fellow students | Skill development through group discussion | |
| | I do discuss because like this is a group we formed so that we can discuss mathematics together every evening before we go home | | |